MMath/JRF Algebraic Topology
Mid-sem test

Instructions : Total time 2 hours. Solve any four problems. All problems carry equal weight. You may use any result done in the class without proof to solve the problems.

1. Let $p : Y \to X$ be a two-sheeted covering, i.e., all fibers have two elements. Prove that this has a unique structure of a G-covering, for $G = S_2$, the symmetric group on two letters.

2. Explain how you would construct a covering space $P : Y \to X$ whose deck transformation group $Aut(Y/X)$ is isomorphic to a given finitely generated abelian group G.

3. Let X be a simply connected space and $p : \widetilde{X} \to X$ is a covering space of X with $\widetilde{X}$ connected. Prove that p must be a homeomorphism.

4. Prove that the unit circle S^1 has an n-sheeted covering space for every $n \geq 1$. Determine the group of deck transformations for such covering spaces.

5. Using Van Kampen theorem compute the fundamental group of the subset $X \subset \mathbb{R}^3$ which is the union of the unit sphere S^2 and a circle that touches S^2 at a point, i.e. X is the wedge of S^2 with a circle.